Levelled Objectives - Calculating

Addition
W

Begin to relate addition to combining two groups of objects.

In practical activities and discussion begin to use the vocabulary involved in adding.

Level 1
Knowledge of Operations and Relationships Between them.
Relate addition to counting on.

Recognise that addition can be done in any order.

Use the vocabulary related to addition and symbols to describe and record addition number sentences.

Facts

Derive and recall all pairs of numbers with a total of 10.

Derive and recall all addition facts for totals to at least 5.

Derive and recall all addition and subtraction facts for each number up to 10.

Mental Methods
Find one more than a number from 1 to 10.
> *Use objects.*
> *Find number on numbered number line then go to the next number.*
> *Put number in head and use counting skills to say the next number.*

Written Methods
Use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two digit number.
> *Use 'Dienes' to add. Record in a number sentence.*

Level 2

<u>Facts</u>

Know all pairs of numbers which total 20.

Use knowledge of addition facts to find all pairs of multiples of 10 that total 100.

Use knowledge of addition facts to find all pairs of multiples of 10 with totals up to 100.

<u>Mental Methods</u>

Add mentally a one-digit number or a multiple of 10 to any two-digit number.

Put larger number first then count on in ones or tens using fingers.

<u>Written Methods</u>

Use practical and informal written methods to add two-digit numbers.

Use a hundred square to count down in tens and then on in ones.
Use 'chips and beans'   34 + 25 = /// .... + //..... = 59
Use 'chips and beans with exchanging   25 + 48 = //..... + ////........
exchange 10 beans for 1 chip then find total by counting tens (chips)   and units (beans).

Use the symbols + and = signs to record and interpret number sentences.

Calculate the value of an unknown in a number sentence. (e.g. □ + 2 = 6, 30 + □ = 44)

Develop and use written methods to record, support or explain addition of two-digit and three-digit numbers.

Use 'waffles, chips and beans.

143 + 235 =   ☐ /////... + ☐☐ ///..... = 378   no exchanging

265 + 183 =   Use waffles, chips and beans with exchanging for tens.

176 + 265 = Use waffles, chips and beans with exchanging for tens and units.

Level 3

Facts

Derive and recall all addition facts for each number to 20.

Use knowledge of addition facts to find sums and differences of multiples of 10 up to 200.

Use knowledge of addition facts and place value to derive sums and differences of pairs of multiples of 10, 100 or 1000. E.g. 70 + 40 =110   700 + 400 = 1100   7000 + 4000 = 11000

Mental Methods

Add mentally combinations of one-digit and two-digit numbers

Put two-digit number in head. Add on one-digit number by jumping (bridging) through 10 or 100.
46 + 8 = 46 + 4 + 4 = 54   98 + 7 = 98 + 2 + 5 = 105
Add 9 by adding 10 then adjusting.
Add 19, 29, 39…by adding a multiple of 10 then adjusting.

Add mentally pairs of two-digit whole numbers (e.g. 47 + 58)

Partitioning Methods
Method 1: *Partition the tens and units: 40 + 7 + 50 + 8.*
Add tens together. 40 + 50 = 90. Add units together 7 + 8 = 15.
Put back together: 90 + 15 = 105

Method 2: *Keep the largest number whole.*
Partition the smaller number. 58 + 40 + 7.
Add on the tens 58 + 40 = 98. Then add on the units 98 + 7 =105

Near Doubles Method
Where two numbers are close together, use doubling and adjusting to add two two-digit numbers.
35 + 36 = double 35 + 1   or   35 + 37 = double 36

Written Methods

Refine and use efficient written methods to add two-digit and three-digit whole numbers and £.p

Column Addition   *Column Addition*   *Column Addition £.p*
HTU + TU   *HTU + HTU*   *£U.th + £U.th*

Estimating and Checking Calculations

Use knowledge of number facts and operations to estimate and check answers to calculations.

Level 4

Mental Methods

Extend mental methods for whole-number calculations.

Add a near multiple of 1000 and adjust. E.g. 450 + 997 = 450 + 1000-3
Look for pairs of numbers that make multiples of 10 e.g. 27 + 36 + 13
27 + 13 = 40   40 + 36 = 76

Calculate mentally with integers and decimals: U.t + U.t

Partitioning methods e.g. 1.4 + 1.7 = 1.7 + 1 + 0.4 = 3.1
Near doubles method e.g. 1.5 + 1.7 = double 1.6
1.5 + 1.6 = double 1.5 + 0.1

Written Methods

Use efficient written methods to add whole numbers and decimals with up to two places.

Use column addition to add 3 digit and 4 digit numbers.
Use column addition to add amounts of money e.g. £421.19 + £349.46.
Use column addition to add 3 numbers e.g. 1202 + 45 + 367

Estimating and Checking Calculations

Use knowledge of rounding, number facts, place value and inverse to estimate and check answers to calculations.

Level 5

Written Methods

Use standard column procedures to add integers and decimals.

Use column method to add several numbers with a differing number of digits. E.g. 4957 +36 +306+42.
Use column method to add decimal numbers that have different numbers of decimal places e.g. 52.85 + 143.6

Subtraction

W
Knowledge of Operations and Relationships Between them.

Begin to relate subtraction to 'taking away'.

In practical activities and discussion begin to use the vocabulary involved in subtracting.

Level 1
Knowledge of Operations and Relationships Between them.

Understand subtraction as 'take away'.

Use the vocabulary related to subtraction and symbols to describe and record subtraction number sentences.

Use the symbols – and = to record and interpret number sentences.

Calculate the value of an unknown in a number sentence. (e.g. 30 – □ = 24)

Mental Methods
Find one less than a number from 1 to 10.
> *Use objects.*
> *Find number on a number line then go back one.*
> *Put number in head then use counting skills to say the number before.*

Find a 'difference'.
> *Build towers using cubes, remove cubes until the towers are the same height.*
> *Count how many cubes have been removed.*
> *Use towers together with a numbered number line: find the largest number then count back to the smaller number.*
> *Use numbered number line without the towers.*

Written Methods
Use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number.
> *Use numbered number line to count back in ones.*
> *Use a hundred square to count back in ones.*

Use practical and informal written methods to support subtraction of a multiple of 10 from a two digit number.
> *Use a hundred square to count back in tens.*

Level 2

Knowledge of Operations and Relationships Between them.

Understand that subtraction is the inverse of addition and vice versa.

Use the inverse to derive and record related addition and subtraction number sentences.

Mental Methods

Subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number.

Put largest number in head. Put smaller number on fingers then count back using fingers. Count in ones or tens.

Written Methods

Use practical and informal written methods to subtract two-digit numbers.

Use a hundred square to count back in tens and then ones.
Use a hundred square together with a blank number line.
e.g.

Develop and use written methods to record, support or explain subtraction of two-digit and three-digit numbers.

Use a blank number line. Use place value to subtract the hundred then use hundred square or two-hundred square if crossing the hundreds barrier. Record on a blank number line.
e.g.   176 - 135

Level 3
Knowledge of Operations and Relationships Between them.

Facts
Derive and recall all subtraction facts for each number to 20.

Use knowledge of subtraction facts to find sums and differences of multiples of 10 up to 200.

Use knowledge of subtraction facts and place value to derive sums and differences of pairs of multiples of 10, 100 or 1000. E.g. 110 – 70 = 40 1100 – 700 = 400   11000 – 7000 = 4000

Mental Methods

Subtract mentally combinations of one-digit and two-digit numbers.
Put larger number in head. Subtract one-digit number by jumping back (bridging) through 10 or 100.
46 - 8 = 46 – 6 – 2 = 38   102 - 7 = 102 – 2 – 5 = 95
Subtract 9 by subtracting 10 then adjusting.
Subtract 19, 29, 39...by subtracting a multiple of 10 then adjusting.

Subtract mentally pairs of two-digit whole numbers (e.g. 58 - 47).
Partitioning Method
Partition the number to be subtracted into tens and units. Subtract the tens then subtract the units. E.g. 58-23 = 58-20-3=38-3=35
Shopkeeper Method
(Counting up as a subtraction technique has NOT been taught before this).
Find a difference by counting up: E.g. 92-89 count up from 89 to 92.
Towers of cubes could be used for the children to see this practically.(Use lower numbers when using cubes!)

Written Methods
Refine and use efficient written methods to subtract two-digit and three-digit whole numbers and £.p

Column Method	*Column Method*	*Column Method*
HTU – TU	*HTU – HTU*	*£ U.th –£U.th (money)*

Estimating and Checking Calculations
Use knowledge of number facts and operations to estimate and check answers to calculations.

Level 4

Mental Methods

Extend mental methods for whole-number calculations.

> *To subtract one near multiple of 100 or 1000 then adjust*
> *e.g. 6070 – 4097 = 6070 – 5000 + 3 = 1073*
>
> *Shopkeeper Method*
> *Use for larger numbers that cross the 100 barrier. E.g. 403-386. Count up from*
> *386 to 400 then count on to 403. 386 + 14 + 3 = 403 so 403 -386 = 17*

Calculate mentally with integers and decimals: U.t - U.t

> *Subtract 0.9, 1.9, 2.9...by subtracting the nearest multiple of 10 then adjusting.*
> *e.g. 4.5 – 0.9 = 4.5 – 1 + 0.1=3.5 +0.1 = 3.6*
> *Partitioning Method*
> *Partition the number to be subtracted into units and tenths. Subtract the units*
> *then subtract the tenths. E.g. 4.8-3.6 = 4.8 – 3 – 0.6 = 1.8 – 0.6 = 1.2*
> *Shopkeeper Method*
> *Find a difference by counting up: E.g. 4.2 – 3.6 count up from 3.6 to 4.2*
> *3.6 + 0.4 + 0.2 = 4.2   so 4.2 – 3.6 = 0.6*

Written Methods

Use efficient written methods to subtract whole numbers and decimals with up to two places.

> *Use written method ThHTU – HTU and ThHTU – ThHTU.*
> *Use written method which requires more than one exchange e.g. 2006 – 247 or*
> *1025 -336 or £20 - £12.45.*

Estimating and Checking Calculations

Use knowledge of rounding, number facts, place value and inverse to estimate and check answers to calculations.

Level 5

Written Methods

Use standard column procedures to subtract integers and decimals.

> *Use column method to subtract decimal numbers that have different numbers of*
> *decimal places e.g. 128.6 – 83.75*

Multiplication

Level 1

Facts
Recall the doubles of all numbers up to double 5.

Mental Methods
Count repeated groups of the same size.
> *Repetitive step counting.*

Level 2

Knowledge of Operations and Relationships Between them.
Represent repeated addition and arrays as multiplication.

Use the symbols x and = to record and interpret number sentences.

Calculate the value of an unknown in a number sentence.(e.g. □ x 2 = 8)

Understand that halving is the inverse of doubling and vice versa.

Solve practical problems that involve combining groups of 2, 5 or 10.

Facts
Recall the doubles of all numbers to at least double 10.

Derive and recall doubles of all numbers to 20.

Derive the multiples of 2, 5 and 10 to the tenth multiple through counting up.

Derive and recall multiplication facts for the 2, 5 and 10 times-tables.

Written Methods
Use practical and informal written methods and related vocabulary to support multiplication.
> *Drawing out beans into groups. E.g. 3 x 5 =*
> *Setting out as an array   ooooo   count in 3s or 5s to total.*
> *ooooo*
> *ooooo*

> *Use chips and beans to draw into groups for larger numbers*

e.g. 24 x 3 =   //....   //....   //....

Add up the chips and beans using exchanging to convert 10 beans into a chip..

Level 3

Knowledge of Operations and Relationships Between them.
Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences.

Facts
Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables.

Derive and recall multiplication facts up to 10 × 10.

Mental Methods
Use knowledge of doubling numbers up to 20 to double multiples of 10 or 100. E.g. use double 7 to calculate double 70.

Double any two digit number.
> *Partitioning Method*
> *Double the tens, double the units then put back to together.*
> *e.g. double 46 = double 40 + double 6 = 80 + 12 = 96*

Multiply one-digit and two-digit numbers by 10 or 100, and describe the effect.

Written Methods
Use practical and informal written methods to multiply two-digit numbers (e.g. 14 × 3)

Grid Method		*HTU*
10	*4*	
3	30	12

> *Draw grid like a noughts and crosses grid without the bottom line. Total the numbers in the grid using column addition. Set out HTU columns ready at the side.*

Estimating and Checking Calculations
Use knowledge of number facts and operations to estimate and check answers to calculations.

Level 4

Knowledge of Operations and Relationships Between them.

Know what a factor of a number means and find all the factors for a given number.

Identify pairs of factors of two-digit whole numbers and find common multiples (e.g. for 6 and 9)

Know what a squared number is.

Facts

Know the squares of numbers to 12x12.

Mental Methods

Double a decimal number.

> *Partitioning Method*
> *Double the units, double the tenths then put back to together.*
> *e.g. double 3.7 = double 3 + double 0.7 = 6 + 1.4 = 7.4*

Extend mental methods for whole-number calculations, for example to multiply a two-digit by a one-digit number (e.g. 14 × 9), to multiply by 25 (e.g. 16 × 25)

> *Partitioning Method*
> *14 x 6 = (10 x 6) + (4 x 6) = 60 + 24 = 84*
>
> *Use doubling and halving*
> *To multiply by 4: Double then double again.*
> *E.g. 34 x 4 = 34 x 2 x 2 = 68 x 2 = 136*
> *To multiply by 5: Multiply by 10 then halve.*
> *E.g. 34 x 5 = 34 x 10 ÷ 2 = 340 ÷ 2 = 170*
>
> *To multiply by 9*
> *Multiply by 10 then adjust.*
> *E.g. 24 x 9 = (24 x 10) – (24 x 1) = 240 – 24 = 216*
>
> *To Multiply by 25*
> *Use knowledge of doubling to deduce multiples of 25 1x25=25, 2x25=50, 4x25=100, 8x25=200, 16x25=400*

Multiply numbers to 1000 by 10 and then 100 (whole-number answers), understanding the effect; relate to scaling up or down.

Use knowledge of multiplication facts to multiply multiples of 10. E.g. use 7 x 8 = 56 to know that 70 x 8 = 560 or 70 x 80 = 5600

Use knowledge of squared numbers to find squares of multiples of 10.

Written Methods

Develop and use written methods to record, support and explain multiplication.

Column method TU x U   *Column Method* HTU × U   Column Method U.t × U

Estimating and Checking Calculations

Use knowledge of rounding, number facts, place value and inverse to estimate and check answers to calculations.

Level 5

Knowledge of Operations and Relationships Between them.
Recognise that prime numbers have only two factors.
Use and understand language including: common factors, highest common factor.

Facts
Know prime numbers up to 100.
Find the prime factors of two-digit numbers.
Use knowledge of times tables to calculate other table facts e.g. x32.

Mental Methods
Use understanding of place value to multiply whole numbers and decimals by 10, 100 or 1000

Calculate mentally with integers and decimals: TU x TU, U.t × U
> *To multiply by 50: multiply by 100 then halve.*
> *e.g. 32 x 50 = 32 x 100 ÷ 2 = 320 ÷ 2 = 160*
> *To multiply by 15: Multiply by 10 then halve the result then add the two parts together.*
>
> *Use knowledge of place value and multiplication facts to 10 × 10 to derive related multiplication facts involving decimals (e.g. 0.8 × 7)*
>
> *Use partitioning to multiply U.t x U*
> *e.g. 1.6 x 7 = (1 x 7) + (0.6 x 7) = 1 + 4.2 = 5.2*
>
> *Use knowledge of doubling two digit numbers and place value to double decimals.*
> *E.g. double 4.6 =((4.6 x 10) doubled) ÷ 10 = (46 doubled) ÷ 10 = 92 ÷ 10 = 9.2*
>
> *Double a decimal less than one using one of the methods above e.g. double 0.15*

Written Methods

Refine and use efficient written methods to multiply TU × TU.

Grid Method

Th   H   T   U

Column Method

Refine and use efficient written methods to multiply HTU x TU

Use column method to multiply HTU x TU

Division

Level 1
Knowledge of Operations and Relationships Between them.
Share objects into equal groups and count how many in each group

Level 2
Knowledge of Operations and Relationships Between them.
Represent sharing as division.

Solve practical problems that involve sharing into equal groups.

Use the symbols ÷ and = to record and interpret number sentences.

Calculate the value of an unknown in a number sentence. (e.g. □ ÷ 2 = 6)

Understand that halving is the inverse of doubling and vice versa.

Facts
Derive and recall the corresponding halves for the doubles of all numbers to 20.

Derive and recall the related division facts for the 2, 5 and 10 times-tables.

Recognise multiples of 2, 5 and 10.

Written Methods
Use sharing with beans. No remainders. Exact multiples only. E.g. 12 ÷ 3

Use sharing with chips and beans. No remainders. Exact multiples only. E.g.69 ÷ 3

Use sharing with chips and beans. No remainders. Multiples only but with splitting of a chip into beans when necessary. E.g. 76 ÷ 2

Level 3
Knowledge of Operations and Relationships Between them.
Represent repeated subtraction as division.

Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences.

Facts
Derive and recall the corresponding division facts for the 2, 3, 4, 5, 6 and 10 times-tables.

Recognise multiples of 2, 5 or 10 up to 1000

Mental Methods
Use knowledge of halving numbers up to 20 to halve multiples of 10 or 100. E.g. Use half of 14 to calculate half of 140.

Halve any two digit number.
> *Partitioning Method*
> *Halve the tens, halve the units then put back to together.*
> *e.g. halve 46 = halve 40 + halve 6 = 20 + 3 = 23*

Written Methods
Use practical and informal written methods and related vocabulary to support division, including calculations with remainders.
> *Use chips and beans sharing with remainders recognising whether or not there are enough left over at the end round to start sharing out again.*
> *E.g. 68 ÷ 3*

Use practical and informal written methods to divide two-digit numbers (e.g. 98 ÷ 6).
> *Bridge Method*
> *Use bridge method. The troll sits underneath the bridge with all of his money. The farmers who want his money stand at the side of the bridge. The farmers defeat the troll and share out his money.*
> *No carrying, no remainders. E.g. 69 ÷ 3*
>
> *No carrying with remainders. E.g. 67 ÷ 3*
>
> *Carrying, no remainders. E.g. 72 ÷ 3*
>
> *Carrying with remainders. E.g. 79 ÷ 3*

Estimating and Checking Calculations
Use knowledge of number facts and operations to estimate and check answers to calculations.

Level 4

Knowledge of Operations and Relationships Between them.
Know and use the term 'common multiple'.

Facts
Derive and recall the corresponding division facts for the multiplication facts up to 10 x 10.

Mental Methods
Divide numbers to 1000 by 10 and then 100 (whole-number answers), understanding the effect; relate to scaling up or down.

Halve a decimal number.
> *Partitioning Method*
> *Halve the units, halve the tenths then put back to together.*
> *e.g. halve 4.6 = halve 4 + halve 0.6 = 2 + 0.3 = 2.3*

Extend mental methods for whole-number calculations.
> *Use doubling and halving*
> *To divide by 4: Halve then halve again.*
> *E.g. 72 ÷ 4 = 72 ÷ 2 ÷ 2 = 36 ÷ 2 = 18*
> *To divide by 5: Divide by 10 then double.*
> *E.g. 170 ÷ 5 = (170 ÷ 10) x 2 = 17 x 2 = 34*

Written Methods
Refine and use efficient written methods to divide HTU ÷ U
> *Use bridge method for HTU ÷ U no carrying and no remainders.*

> *Use bridge method for HTU ÷ U no carrying but remainders.*

> *Use bridge method for HTU ÷ U carrying but no remainders.*

> *Use bridge method for HTU ÷ U carrying and remainders.*

Estimating and Checking Calculations
Use knowledge of rounding, number facts, place value and inverse to estimate and check answers to calculations.

Level 5

<u>Knowledge of Operations and Relationships Between them.</u>
Know what the square root of a number is.
Use and understand language including: lowest common multiple, highest common multiple.

<u>Facts</u>
Recognise the square roots of perfect squares to 12x12.

<u>Mental Methods</u>
Use understanding of place value to divide whole numbers and decimals by 10, 100 or 1000.

Calculate mentally with decimals: U.t ÷ U.
> *Use knowledge of place value and multiplication facts to 10 × 10 to derive related division facts involving decimals (e.g. 4.8 ÷ 6)*

Extend mental methods to divide mentally.
> *Use knowledge of halving two digit numbers and place value to halve decimals.*
> *E.g. halve 4.6 =((4.6 x 10) halved) ÷ 10 = (46 halved) ÷ 10 = 23 ÷ 10 = 2.3*
>
> *Halve a decimal less than one using one of the methods above e.g. halve 0.15.*

<u>Written Methods</u>
Refine and use efficient written methods to divide U.t ÷ U
Use bridge method for U.t ÷ U with no carrying and no remainders. E.g. 9.6 ÷ 3

Use bridge method for U.t ÷ U with carrying but no remainders. E.g. 5.1 ÷ 3

Use bridge method for U.t ÷ U using additional decimal places to share out remainders. E.g. 9.3 ÷ 6

Use standard column procedures to divide three-digit integers by a two-digit integer. HTU ÷ TU

Chunking Method
Repeatedly subtract amounts from the original amount.
Without remainders.

With remainders

Solve a problem writing the answer as a mixed number. (Write the quotient as a fraction).

Estimating and Checking Calculations
Use tests of divisibility to estimate and check answers to calculations.
Make and justify estimates and approximations to calculations.